# MATHEMATICAL MODELING OF EXPLOSIVE PROCESSES IN INHOMOGENEOUS STARS

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**Abstract.** The work considers a non-self-similar problem about the central explosion of nonhomogeneous gas body (star) bordering vacuum which is in equilibrium in its own gravitational field. To solve the problem, the asymptotic method of thin impact layer has been used. The solution of the problem in the vicinity behind the shock wave (the destruction surface of the first kind) is sought in the form of a singular asymptotic decomposition by a small parameter. Analytically, the main (zero) approximation for the law of motion and the thermodynamic characteristics of the medium has been accurately found. The Cauchy problem for zero approximation of the law of motion of the shock has been solved exactly, in the form of elliptic integrals of the first and second general ones. The relevant asymptotics have been found.

***Keywords:*** *Nonhomogeneous star, gravitational field, explosion, shock wave, singular decomposition.*

**Introduction.** Mathematical modeling of explosive processes in gravitational gaseous bodies is one of the actual problems in astrophysics [1-10].

According to the existing understanding, the light elements contained in the outer layers of the stars can detonate. Detonation is initiated during gravitational collapse of a gas nucleus accompanied by neutron radiation. The main focus is on the physical processes associated with thermonuclear reactions and the propagation of radiation, and less attention is paid to the dynamics of gases, on the whole [4].

Due to the complexity of the problem, numerical modeling of these processes has been widely developed [5].

In modern astrophysics the catastrophic processes of stellar explosion with subsequent formation of neutron stars and collapsing bodies - black holes - are of special interest. Novae and supernovae explosions are the non-stationary motions of large masses of gas with sharply increasing radiation energy.

In order to solve number of problems of astrophysics it is necessary to study the dynamics of gaseous bodies interacting with the gravitational field. In this regard, the problems of the process of propagation of explosive detonation waves in the gravitational field are worth noting.

It is clear that the study of astrophysical phenomena should be based on setting and solution of number of dynamic problems of gas movement considered as mathematical models with important features of stellar movement and evolution.

For theoretical understanding of such grand cosmic catastrophes, application of the methods of dimension theory to the problems of gas dynamics allowed L. Sedov to propose and solve number of classical problems from the theory of point explosion. Such problems are connected with the movement of shock waves and free surfaces. These works laid the groundwork for the development of the great direction of modern gas dynamics - the Big Explosion theory, the problems of which usually require application of complex computational methods [1].

Basically, the essential and practically important parameter of these problems is the law of motion of the shock wave generated by explosion, but in terms of differential equations the classical formulation of the problem usually involves the predetermination of the properties of the whole local process. On the other hand, description of the explosion phenomena in an ideal mathematical model requires a certain accuracy of calculation. In this regard, obviously it would be important to approximately define the sought unit of the integral characteristic by means of estimation of the systems of inequalities allowing us to obtain simple two-sided estimations for it. In many cases these estimations are sufficient to solve it [6].

Very often small reduction of exact estimations allows us to fully express the answer in elementary functions. The conclusion and solution of necessary inequalities represent the development of the method of integral relation, well-known in hydrodynamics [1].

Based on the equations of the motion of medium, the integral equations of energy and Lagrange-Jacobi for one-dimensional spherical-symmetric flows of perfect gravitational gas T. Chilachava [6] solved a system of integro-differential inequalities for the law of motion of the detonation wave and the moment of inertia of the disturbed area. For justification of the solution of initial inequalities and their simplification in the case of motion of detonate wave in practically quiet gas a system of inequalities is obtained and given in its final form using the Helder and Jensen theorems (of inequalities). A self-similar spherically symmetric problem of adiabatic motion of gravitating perfect gas is considered for a detonation wave occurring in case of collapse of a nonhomogeneous gravitational gas at zero pressure or in case of imbalance. Using the method of integrodifferential inequalities, a system is obtained that determines the law of motion of the detonation wave, based on the known initial state of the gas.

In [11-14] T. Chilachava proposed an asymptotical method for the gravitating perfect gas, which is connected with small parameter $\varepsilon = \frac{\gamma_2 - 1}{\gamma_2 + 1}$ (the asymptotic thin-shock-layer method). By means of this method the non-similar problem of propagation of detonation wave exploding in the nucleus balanced in its own gravitational field is found. The adiabatic expansion of a gaseous body into a vacuum is described.

The problem of stationary solid-state rotation with a constant angular speed of a homogeneous three-axis gas ellipsoid being in its own gravitational field bounded by a vacuum is discussed in [15, 16].

As is well known, in the equilibrium theory of ellipsoidal figures, Jacob's three-axial steady-state rotational half-axes as well as the angular speed of rotation must satisfy some additional relations. Partial derivative equations of gravitational gas motion (vector equation of gas motion, entropy and scalar equations of continuity) are considered in both Euler (Cartesian) and spherical coordinates suitable for solving such types of problems. An exact solution (the law and speed of the motion of the medium and also thermodynamic characteristics of the medium) to the problem of stationary solid-state rotation with a constant angular speed of a homogeneous three-axis gas ellipsoid, which is in its own gravitational field and is bounded by a vacuum (zero pressure on boundary) is found. The distribution of gravitational field potential in a three-axis homogeneous ellipsoid satisfying Poisson's equation is also found.

***Chapter I. The system of equations and boundary conditions of spherically symmetric* motion *of gravitating gas on the surface of a strong rupture***

In the Lagrangian coordinates the system equations and conditions of adiabatic spherically symmetric motion of gravitating perfect gas on the surface of a strong rupture are given by:

$$\frac{\partial^2 r}{\partial t^2} + 4\pi r^2 \frac{\partial p}{\partial m} + \frac{km}{r^2} = 0,\ p = (\gamma - 1) f(m) p^{\gamma},\ \rho = \left[4\pi r^2 \frac{\partial r}{\partial m}\right]^{-1}, \tag{1.1}$$

$$[r]_1^2 = \left[v\dot{M} - 4\pi r^2 p\right]_1^2 = 0, \tag{1.2}$$

$$\left[\dot{M}\left(\frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2 + \frac{p}{(\gamma-1)\rho} - \frac{kM}{R}\right) - 4\pi r^2 v p\right]_1^2 = 0,$$

where $r, p, \rho \| m, t,$ are the unknown functions; $\frac{\partial r}{\partial t} = v$ is the speed of gas motion; $k$- gravitation constant, and $f(m)$ function is connected to the entropy distribution by the Lagrangian $m$ coordinate.

The equation of motion $t$ in (1.1) is obtained from the equation of motion t in the Euler coordinates

$$\frac{dv}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{\partial \Phi}{\partial r}, \tag{1.3}$$

where $\Phi$ is the potential of the gravitation field defined from the equation:

$$\Delta\Phi = -4\pi k\rho. \tag{1.4}$$

In Cartesian coordinates, the Laplacian is given by:

$$\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, r = \sqrt{x^2 + y^2 + z^2}.$$

Write the Laplacian in spherical coordinates $(x, \gamma, z) \Rightarrow (r, \theta, \varphi)$,

$$\begin{cases} x = rsin\theta cos\theta \\ y = rsin\theta cos\varphi \\ z = rcos\theta \end{cases} \Rightarrow \begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \varphi = arctg\frac{y}{x} \\ \theta = arccos\frac{z}{r} \end{cases},$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial}{\partial \varphi}\frac{\partial \varphi}{\partial x} + \frac{\partial}{\partial \theta}\frac{\partial \theta}{\partial x} = \frac{\partial}{\partial r}\frac{x}{r} + \frac{\partial}{\partial \varphi}\frac{-y}{x^2+y^2} + \frac{\partial}{\partial \theta}\frac{xz}{r^2\sqrt{x^2+y^2}},$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial}{\partial \varphi}\frac{\partial \varphi}{\partial y} + \frac{\partial}{\partial \theta}\frac{\partial \theta}{\partial y} = \frac{\partial}{\partial r}\frac{y}{r} + \frac{\partial}{\partial \varphi}\frac{x}{x^2+y^2} + \frac{\partial}{\partial \theta}\frac{yz}{r^2\sqrt{x^2+y^2}},$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial}{\partial \varphi}\frac{\partial \varphi}{\partial z} + \frac{\partial}{\partial \theta}\frac{\partial \theta}{\partial z} = \frac{\partial}{\partial r}\frac{z}{r} + \frac{\partial}{\partial \theta}\frac{-(x^2+y^2)}{r^2\sqrt{x^2+y^2}},$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial r^2}\frac{x^2}{r^2} + \frac{\partial^2}{\partial \varphi^2}\frac{y^2}{(x^2+y^2)^2} + \frac{\partial^2}{\partial \theta^2}\frac{x^2z^2}{r^4(x^2+y^2)} + \frac{\partial^2}{\partial r\partial\varphi}\frac{-2xy}{r(x^2+y^2)} + \frac{\partial^2}{\partial r\partial\theta}\frac{2x^2z}{r^3\sqrt{x^2+y^2}} +$$
$$+ \frac{\partial^2}{\partial\varphi\partial\theta}\frac{-2xyz}{r^2\sqrt{(x^2+y^2)}^3} + \frac{\partial}{\partial r}\frac{y^2+z^2}{r^3} + \frac{\partial}{\partial\varphi}\frac{2xy}{(x^2+y^2)^2} +$$
$$+ \frac{\partial}{\partial\theta}\frac{z[r^2(x^2+y^2) - x^2(3(x^2+y^2)+z^2)]}{r^4\sqrt{(x^2+y^2)}^3},$$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial r^2}\frac{y^2}{r^2} + \frac{\partial^2}{\partial \varphi^2}\frac{x^2}{(x^2+y^2)^2} + \frac{\partial^2}{\partial \theta^2}\frac{y^2z^2}{r^4(x^2+y^2)} + \frac{\partial^2}{\partial r\partial\varphi}\frac{2xy}{r(x^2+y^2)} + \frac{\partial^2}{\partial r\partial\theta}\frac{2y^2z}{r^3\sqrt{x^2+y^2}} +$$
$$+ \frac{\partial^2}{\partial\varphi\partial\theta}\frac{2xyz}{r^2\sqrt{(x^2+y^2)}^3} + \frac{\partial}{\partial r}\frac{x^2+y^2}{r^3} + \frac{\partial}{\partial\varphi}\frac{-2xy}{(x^2+y^2)^2} +$$
$$+ \frac{\partial}{\partial\theta}\frac{z[r^2(x^2+y^2) - y^2(3(x^2+y^2)+z^2)]}{r^4\sqrt{(x^2+y^2)}^3},$$

$$\frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial r^2}\frac{z^2}{r^2} + \frac{\partial^2}{\partial \theta^2}\frac{(x^2+y^2)}{r^4} + \frac{\partial^2}{\partial r\partial\theta}\frac{-2z(x^2+y^2)}{r^3\sqrt{x^2+y^2}} + \frac{\partial}{\partial r}\frac{x^2+y^2}{r^3} + \frac{\partial}{\partial\theta}\frac{2z\sqrt{x^2+y^2}}{r^4}.$$

Finally, we get:

$$\Delta \equiv \Delta \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2sin^2\theta}\frac{\partial^2}{\partial\varphi^2}. \tag{1.5}$$

But in the case of spherically symmetric motion:

$$\frac{\partial}{\partial\theta}=\frac{\partial}{\partial\varphi}=0,$$

Therefore, from (1.4), (1.5) we will have:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)=-4\pi k\rho, \tag{1.6}$$

$$\frac{\partial\Phi}{\partial r}=-\frac{1}{r^2}4\pi k\int_0^r\rho z^2dz+\frac{c_1}{r^2}, \Phi=-4\pi k\int_0^r\frac{1}{r^2}\left(\int_0^r\rho z^2dz\right)dr-\frac{c_1}{r}+c_2,$$

$$c_1=0,\ c_2=0,$$

$$\frac{\partial\Phi}{\partial r}=\frac{-4\pi k\int_0^r\rho z^2dz}{r^2}. \tag{1.7}$$

In the latter relation the integral

$$4\pi\int_0^r\rho z^2dz=m, \tag{1.8}$$

and from (1.3) we easily get the equation of the gravitating gas motion in the form of (1.1).

Thus, $r=r(m,t)$ is the law of motion of medium; $m=M(t)$ - the law of motion of the surface of strong rupture in Lagrange mass coordinate, and $R=r(M(t),t)$ - the law of movement of the surface of strong rupture (radius).

The integral equation of energy is given by:

$$T+U-kV=E+\int_0^t\left\{\dot{M}\left[\frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2+\frac{p}{(\gamma-1)\rho}-\frac{kM}{R}\right]-4\pi r^2\frac{\partial r}{\partial t}p\right\}_1d\tau, \tag{1.9}$$

$$T=\frac{1}{2}\int_0^M\dot{r}^2dm, =\frac{1}{\gamma-1}\int_0^M\frac{P}{\rho}dm, V=-\int_0^M\frac{mdm}{r}.$$

***Chapter II. Exact Solution to the Initial Shock Wave***

Now let the exact solution to (1.1) be the initial data corresponding to a nonhomogeneous gas nucleus bounded by a vacuum balanced in its own gravitational field, where the gravitational constant $k$, the density at the center of the nucleus $\rho_0$ and the radius of the nucleus are the main units of dimension.

Let the distribution of density of a nonhomogeneous gravitating gas nucleus be given by (in non-dimensional form):

$$\rho=1-r, \tag{2.1}$$

which shows that the density is maximum at the center of the nucleus, and it is zero on the sphere (the star is bounded by the interstellar medium the density of which is $\rho\sim10^{-24}gr/cm/cm^3$ or actually it is zero).

Introduction of the density distribution (2.1) into formula (1.8) of Lagrange coordinate $m$ gives:

$$m = 4\pi r^3\left(\frac{1}{3} - \frac{r}{4}\right). \tag{2.2}$$

Because the nucleus in initial state is balanced in its gravitational field

$$\frac{\partial r}{\partial t} = v = 0. \tag{2.3}$$

Introduction of (2.3), (2.2), (2.1), (1.7) into (1.3) taking into consideration that the pressure on the sphere is zero (actually the star is bounded by a vacuum) gives the distribution of pressure in the nucleus (in non-dimensional form):

$$p = 4\pi\left[\frac{1}{6}(1 - r^2) - \frac{7}{36}(1 - r^3) + \frac{1 - r^4}{16}\right]. \tag{2.4}$$

Thus, in the form of the initial data (initial exact solution before the shock wave), we receive the exact solution of the system (1.1):

$$\rho = 1 - r, m = 4\pi r^3\left(\frac{1}{3} - \frac{r}{4}\right), \tag{2.5}$$

$$\frac{\partial r}{\partial t} = v = 0,$$

$$p = 4\pi\left[\frac{1}{6}(1 - r^2) - \frac{7}{36}(1 - r^3) + \frac{1 - r^4}{16}\right].$$

***Chapter III. Approximate Analytical Solution to the after Shock Waves***

The conditions of the rupture of the first kind (1.2), which are solved after the shock wave with respect to the parameters (unknown functions) are written as follows:

$$\rho_2 = \frac{\gamma+1}{\gamma-1}\rho_1\left[1 + \frac{1}{\gamma-1}\frac{2a_1^2}{\left(\dot{R} - \left(\frac{\partial r}{\partial t}\right)_1\right)^2}\right]^{-1}, a_1^2 = \frac{\gamma p_1}{\rho_1},$$

$$p_2 = \frac{1}{\gamma+1}\left[p_1(1-\gamma) + 2\rho_1\left(\dot{R} - \left(\frac{\partial r}{\partial t}\right)_1\right)^2\right], \tag{3.1}$$

$$\dot{R} - \left(\frac{\partial r}{\partial t}\right)_2 = \frac{1}{\gamma+1}\left(\dot{R} - \left(\frac{\partial r}{\partial t}\right)_1\right)\left[\gamma - 1 + \frac{2a_1^2}{\left(\dot{R} - \left(\frac{\partial r}{\partial t}\right)_1\right)^2}\right].$$

Here, the continuity of the Euler and Lagrange coordinates must be taken into consideration.

$$[r]_1^2 = 0, [m]_1^2 = 0. \tag{3.2}$$

Thus, we obtain the initial-boundary (mixed) problem for the functions $r(m,t), p(m,t), \rho(m,t)$ of unknown system (1.1) of nonlinear, nonhomogeneous partial differential equations.

The initial conditions (2.5) determine the initial state of the nonhomogeneous gravitational gas nucleus and represent the exact solution to the system (1.1) of equations.

Thus, we consider the initial-boundary problem in $\Omega$ area:

$$\Omega = \{t \in (0, t_*), m \in (0, M(t))\}, \tag{3.3}$$

where $t = 0$ is the moment of explosion, $t_*$ - the moment of time when the shock wave appears on the surface of the body (sphere).

The boundary conditions of an unknown boundary $m = M(t)$ are given by (3.1), and at the center of symmetry

$$r = 0, m = 0. \tag{3.4}$$

To find an approximate asymptotic solution to a mixed problem, apply the asymptotic method of the thin shock layer thar was introduced by Academician G. Chorny for a perfect non-gravitatingl gas, and was first proposed by T. Chilachava for gravitating gas and is connected to a small parameter

$$\varepsilon = \frac{\gamma-1}{\gamma+1}. \tag{3.5}$$

To use this method, the magnitude of the explosion energy must be relative to the initial state parameters of $\frac{1}{\varepsilon^2}$ order before the shock wave strikes the surface of nucleus.
Assume that asymptotics of L. Sedov's solution to the powerful point explosion [1] is realized in the basic approximation of the law of motion of medium.

Analysis of the condition of existence of the strong shock wave before appearing on the surface, and the integral equation (1.9) of energy leads to the condition

$$E = \frac{E_0}{\varepsilon^2},\ E_0 = O(1). \tag{3.6}$$

Here, the time necessary for the shock wave to appear on the surface will be of $\varepsilon^{\frac{1}{2}}$ order. Therefore, it is convenient to perform an additional extension of time

$$\tau = t/\sqrt{\varepsilon}. \tag{3.7}$$

The analysis of equation (1.1) of motion and the boundary conditions (3.1), and (3.6), (3.7) shows that the approximate solution to (1.1) can be sought in the after shock wave by means of a small parameter (3.5) given by the following singular decomposition:

$$r = R_0(\tau) + \varepsilon H(m,\tau) + \ldots, R(\tau) = R_0(\tau) + \varepsilon R_1(\tau) + \ldots, \tag{3.8}$$

$$p = p_0(m,\tau) + \varepsilon p_1(m,\tau) + \ldots, \rho = \frac{\rho_0(m,\tau)}{\varepsilon} + \rho_1(m,\tau) + \ldots.$$

Introducing the singular decomposition (3.8) into the system (1.1) of equations, integral equation (1.9) and the boundary conditions (3.1), we get the zero approximation of the solution to the problem**:**

$$p_0(m,\tau) = R_0'^2(\tau)\big(1 - R_0(\tau)\big) + \frac{R_0''(\tau)(M_0(\tau) - m)}{4\pi R_0^2(\tau)},$$

$$M_0(\tau) = 4\pi R_0^3(\tau)\left(\frac{1}{3} - \frac{R_0(\tau)}{4}\right), \tag{3.9}$$

$$\rho_0(m,\tau) = p_0^{1/\gamma}(m,\tau)\left[R_0'^2(T_0(m))\right]^{-1/\gamma}\left[1+\frac{a_1^2(m)}{\gamma R_0'^2(T_0(m))}\right]^{-1},$$

$$a_1^2(m) = \frac{4\pi\gamma}{1-r}\left[\frac{1}{6}\left(1-r^2(m)\right)-\frac{7}{36}\left(1-r^3(m)\right)+\frac{1-r^4(m)}{16}\right],$$

where $r = r(m)$ is defined from equation (2.2)

$$m = 4\pi r^3\left(\frac{1}{3}-\frac{r}{4}\right),$$

$$r^4 - \frac{4}{3}r^3 + \frac{m}{\pi} = 0,$$

$$r \in [0;1], m \in \left[0;\frac{\pi}{3}\right].$$

And is given by:

$$r(m) = \frac{1}{3} + \frac{\sqrt{2(A+B)+\frac{4}{9}} - \sqrt{\frac{8}{9}-2(A+B)+\frac{16}{27\sqrt{2(A+B)+\frac{4}{9}}}}}{2},$$

$$A(m) = \sqrt[3]{\frac{m}{9\pi}\left(1+\sqrt{1-\frac{3m}{\pi}}\right)}, \tag{3.10}$$

$$B(m) = \sqrt[3]{\frac{m}{9\pi}\left(1-\sqrt{1-\frac{3m}{\pi}}\right)}.$$

In (3.9), $T_0 = T_0(m)$ is the time moment, when the shock wave passes the particle through the Lagrange coordinate $m$.

The function $R_0(\tau)$ in (3.9) is the solution to the following Cauchy problem:

$$\pi[1-R_0(\tau)]R_0'^2 R_0^3(\tau)\left[\frac{1}{3}-\frac{R_0(\tau)}{4}\right] = E_0, R_0(0) = 0, \tag{3.11}$$

$$\int_0^{R_0}\sqrt{(1-R_0)(4-3R_0){R_0}^3}\,dR_0 = \int_0^{\tau}\sqrt{\frac{12E_0}{\pi}}\,d\tau,$$

$$\int_0^{R_0}\sqrt{(1-R_0)(4-3R_0){R_0}^3}\,dR_0 = \sqrt{\frac{12E_0}{\pi}}\,\tau,$$

$$\int_0^{1}\sqrt{(1-R_0)(4-3R_0){R_0}^3}\,dR_0 = \sqrt{\frac{12E_0}{\pi}}\,\tau_*, \tag{3.12}$$

$\tau_*$ is the time moment, where the shock wave appears on the nucleus (star) surface.

$$\int_0^x x\sqrt{(1-x)(4-3x)x}\,dx =$$

$$= \frac{\mathbf{2\left\{160\sqrt{x(1-x)(4-3x)}\,F\left[\sin^{-1}\left(\frac{2}{\sqrt{3x}}\right)\middle|\frac{3}{4}\right]-616\sqrt{x(1-x)(4-3x)}\,E\left[\sin^{-1}\left(\frac{2}{\sqrt{3x}}\right)\middle|\frac{3}{4}\right]+y(x)\right\}}}{\mathbf{2835\sqrt{x(1-x)(4-3x)}}}, \tag{3.13}$$

$$y(x) \equiv 1215x^5 - 3402x^4 + 2259x^3 - 84x^2 + 1244x - 1232.$$

Elliptic integral of the first kind

$$F(\varphi,k) = \int_0^{\varphi}\frac{dt}{\sqrt{1-k^2\sin^2 t}} = \int_0^{\sin\varphi}\frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}},$$

Elliptic integral of the second kind

$$(\varphi, k) = E(\varphi|k^2) = E(sin\varphi; k) = \int_0^{\varphi} \sqrt{1 - k^2 sin^2\theta}\, d\theta,$$

$$\int_0^1 x\sqrt{(1-x)(4-3x)x}\, dx = 0{,}28287. \tag{3.14}$$

From (3.12), (3.14) we get

$$\tau_* = 0{,}28287\sqrt{\frac{\pi}{12E_0}}. \tag{3.15}$$

Find the asymptotics of the solution to the Cauchy problem (3.11) for:

$$\tau \to 0_+, \tau \to \tau_{*-},$$

From (3.11) we get

$$\pi {R'_0}^2 {R_0}^3(\tau) * \frac{1}{3} \cong E_0, \tau \to 0_+,$$

$$\mathrm{R'_0}^2 \mathrm{R_0}^3(\tau) \cong \frac{3E_0}{\pi}, \mathrm{R'_0} \mathrm{R_0}^{3/2}(\tau) \cong \sqrt{\frac{3E_0}{\pi}},$$

$$\int_0^{\mathrm{R_0}} \mathrm{R_0}^{3/2} dR_0 \cong \int_0^{\tau} \sqrt{\frac{3E_0}{\pi}} d\tau, \frac{2}{5}\mathrm{R_0}^{5/2} \cong \sqrt{\frac{3E_0}{\pi}}\tau, \mathrm{R_0}^{5/2} \cong \frac{5}{2}\sqrt{\frac{3E_0}{\pi}}\tau,$$

$$\mathrm{R_0}(\tau) \cong \left(\frac{75E_0}{4\pi}\right)^{\frac{1}{5}} \tau^{2/5},\ \tau \to 0_+. \tag{3.16}$$

for $\tau \to \tau_*, R_0 \to 1-$

$$\pi {R'_0}^2 (1 - R_0) * \frac{1}{12} \cong E_0, \mathrm{R_0}(\tau_*) = 1,$$

$$\mathrm{R'_0}^2(1 - \mathrm{R_0}) \cong \frac{12\mathrm{E_0}}{\pi}, \mathrm{R'_0}\sqrt{(1 - \mathrm{R_0})} \cong \sqrt{\frac{12\mathrm{E_0}}{\pi}},$$

$$\int_{\mathrm{R_0}}^1 \sqrt{(1 - \mathrm{R_0})}\, d\mathrm{R_0} \cong \int_{\tau}^{\tau_*} \sqrt{\frac{12\mathrm{E_0}}{\pi}} d\tau,\ \frac{2}{3}(1 - \mathrm{R_0})^{3/2} \cong \sqrt{\frac{12\mathrm{E_0}}{\pi}}(\tau_* - \tau),$$

$$(1 - \mathrm{R_0})^{3/2} \cong \frac{3}{2}\sqrt{\frac{12\mathrm{E_0}}{\pi}}(\tau_* - \tau),$$

$$\mathrm{R_0}(\tau) \cong 1 - \left(\frac{27\mathrm{E_0}}{\pi}\right)^{\frac{1}{3}} (\tau_* - \tau)^{2/3}, \tau \to \tau_*. \tag{3.17}$$

**Conclusion.** Thus, non-similar problem of the central explosion of an inhomogeneous gaseous body (star) bounded by a vacuum that is balanced in its own gravitational field (the exact solution in the initial shock wave, (2.5)). The asymptotic method of the thin shock layer is used to solve the problem. The solution to the problem is sought in the after shock wave (the rupture of surface of the first kind) in a small parameter in the form of a singular asymptotic decomposition (3.8). The exact basic (zero) approximation of the law of motion of medium and the thermodynamic parameters is analytically found (3.9). Cauchy's problem for the zero approximation of the shock wave law is exactly solved analytically in the form of elliptical integrals of the first and second kind. Relevant asymptotics (3.16), (3.17) are found.

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