RESEARCH OF NON-LINEAR DYNAMIC SYSTEM DESCRIBING INTERACTION BETWEEN COLCHIAN-GEORGIAN AND SVAN POPULACES

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Presented by the Institute of Mathematics named after Academician Ilia Vekua of the Tskhum-Abkhazian Academy of Sciences

Abstract. This paper discusses the two stages of transformation of the Common-Kartvelian-speaking people: the first stage was 5000–2500 BC, when the entire populace spoke the Common Kartvelian language and lived in the South Caucasus; the second stage was 2500–1000 BC, when the entire populace was divided into three parts: Svan, Colchian-Georgian and the third part, Pelasgian tribe, was emigrated to various areas of the European continent. For the first stage, computer simulation is used in the case of variable coefficients of the equation and corresponding numerical values are obtained. The second stage is described by two different mathematical models: one part of the Common-Kartvelian-speaking people went to

Europe and the process of their partial or complete assimilation on the European continent began. Two other parts of the populaces speaking on the Colchian-Georgian and Svan languages that arose as a result of the transformation of the Common-Kartvelian-speaking populace remained in the South Caucasus and Anatolia. To describe the process of interaction between the Colchian-Georgian and Svan peoples, a two-dimensional non-linear system of ordinary differential equations with variable coefficients is proposed. A special case of a two-dimensional system of ordinary differential equations with constant coefficients is considered. In two cases of certain interdependencies between the constant coefficients of the system, it is shown that the divergence of an unknown vector function in the physically significant first quarter of the phase plane changes sign when passing through any segment or half-line one. Taking into account Bendixson's criterion, theorems were proved about the variability of the divergence of a vector field and the existence of closed trajectories in some simply connected domain that completely contains the starting point lying on one segment or half-line. Thus, it is shown that there is no assimilation of the Svan populace by the Colchian-Georgian populace and these two indigenous inhabitants peacefully coexist in the same territory after the transformation of the Common-Kartvelian-speaking populace.

Keywords: *Mathematical modeling, dynamic system, Colchian-Georgian and Svan populaces, Bendixson's criterion.*

Introduction. According to historical and linguistic evidence, the Caucasian tribes, inhabited a large area and had a significant influence on the political map of the world at the time. The development of synergetics gave a powerful push using of mathematical and computer models in social sciences. Mathematical modeling of social processes compared to modeling in natural science is more original due to the complexity of model justifications [1–16]. From a historical point of view, we see mathematical modeling as an innovative approach to describe the area of distribution of the Common-Kartvelian-speaking populace and the process of further transformation of the language, determining the number of the populace speaking the corresponding language in each time period. Mathematical modeling of the first stage is considered in [17].

1. COMPUTER MODELING OF THE FIRST STAGE

Computer modeling for various functions of coefficients gives two qualitatively various pictures, but both at the end of the first stage (2500 BC) yield approximately identical result, the Common-Kartvelian-speaking populace in this century became the 3–3.5 million. The qualitative difference is that: in the first case, the Common-Kartvelian-speaking populace always grows from 1 million to 3–3.5 million; in the second case: the Common-Kartvelian-speaking populace at first it grows to 4–4.5 million (maximum development and economic situation), then, due to certain reasons (perhaps there was not enough land, food, etc.), it began to decrease to 3–3.5 million.

Certain historical sources and mathematical logic says that the second case when the Common-Kartvelian-speaking populace so far as much as possible increased, developed historically took place, economically amplified, then after achievement of a maximum at this level found it difficult to remain, slowly owing to various reasons the easing began (an insufficient area, food problems for such number of the populace, strengthening of the next people, oppressions with their sides and others).

2. MATHEMATICAL MODELING OF THE SECOND STAGE. RESEARCH OF A NONLINEAR DYNAMICAL SYSTEM

The second stage of transformation of the Common-Kartvelian-speaking populace is described by two different mathematical models. The first part of the Common-Kartvelian-speaking people emigrated to Europe and gradually completely or partially assimilated on the European continent. An exact analytical solution was found. Cases of complete or partial Common-Kartvelian-speaking populace assimilation researched.

Two other parts of the populaces speaking on the Colchian-Georgian and Svan languages that arose as a result of the transformation of the Common-Kartvelian-speaking populace remained in the South Caucasus and Anatolia.

To describe the process of interaction between the Colchian-Georgian and Svan populaces, consider a two-dimensional non-linear system of ordinary differential equations with variable coefficients [18]

$$\begin{cases} \frac{dw(t)}{dt} = \alpha_1(\mathrm{t})w(t) - \gamma_1(\mathrm{t})w^2(t) + \beta_1(t)w(t)u(t) - q(\mathrm{t})w(t) \\ \frac{du(t)}{dt} = \alpha_2(\mathrm{t})u(t) - \gamma_2(\mathrm{t})u^2(t) - \beta_2(t)w(t)u(t) \end{cases}, \quad (1)$$

$$w(t_1) = w_1, u(t_1) = u_1, \quad (2)$$

$$t \in (t_1;\ t_2), w(t), u(t) \in C^1[t_1, t_2],$$

$$\beta_i(\mathrm{t}) > 0, \gamma_i(\mathrm{t}) \geq 0, i \in 1{,}2, q(\mathrm{t}) > 0, \alpha_i(\mathrm{t}), \beta_i(\mathrm{t}), \gamma_i(\mathrm{t}), q(\mathrm{t}) \in C[t_1, t_2],$$

$w(t)$ is the number of Colchian-Georgian-speaking populace at t time; $u(t)$ is the number of the populace speaking on the Svan language at the t time; t_1 – 2500 BC, t_2 – 1000 BC, $\alpha_1(\mathrm{t}), \alpha_2(\mathrm{t})$ – natural demographic factor, respectively, of the Colchian-Georgian and Svan populaces; $\gamma_1(\mathrm{t}), \gamma_2(\mathrm{t})$ – co-factors of self-limiting growth, respectively, of the Colchian-

Georgian and Svan populaces; $\beta_1(\mathrm{t}), \beta_2(\mathrm{t})$ – co-factors of assimilation of the Svan populace by the Colchian-Georgian populace; $q(\mathrm{t}) > 0$ – co-factors of unnatural reduction of the Colchian-Georgian populace due to forced hostilities with neighboring peoples.

Detailed qualitative analysis of the system of ordinary differential equations (1), taking into account the adequacy and non-triviality of the mathematical model leads to a system of restrictions on the variable coefficients of the dynamic system

$$\begin{cases} \alpha_2(t) > 0 \\ \gamma_1(\mathrm{t}) \geq 0 \\ \gamma_2(\mathrm{t}) \geq 0 \\ \beta_1(t) > 0 \\ \beta_2(t) > 0 \\ q(\mathrm{t}) > 0 \end{cases}, \qquad t \in [t_1; t_2], \tag{3}$$

For a qualitative analysis of the dynamical system (1), (2), consider a special case, when all coefficients of the system of ordinary differential equations are constants.

$$\alpha_1(\mathrm{t}) = \alpha_1 = const,\ \alpha_2(\mathrm{t}) = \alpha_2 = const,\ \gamma_1(\mathrm{t}) = \gamma_1 = const,\ \gamma_2(\mathrm{t}) = \gamma_2 = const, \tag{4}$$
$$\beta_1(\mathrm{t}) = \beta_1 = const,\ \beta_2(\mathrm{t}) = \beta_2 = const,\ q(\mathrm{t}) = q = const.$$

Non-linear system of ordinary differential equations (1), (2) shall be rewritten in vector form

$$\frac{d\vec{H}(t)}{dt} = \vec{F}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix},\ \vec{H}(\mathrm{t}) = \begin{pmatrix} w(\mathrm{t}) \\ u(t) \end{pmatrix},\ \vec{H}(t_1) = \begin{pmatrix} w_1 \\ u_1 \end{pmatrix}, \tag{5}$$

where according to the system (1), (5) we have

$$F_1\big(w(\mathrm{t}), u(\mathrm{t})\big) = \alpha_1 w(\mathrm{t}) - \gamma_1 w^2(t) + \beta_1 w(\mathrm{t})u(t) - qw(\mathrm{t}), \tag{6}$$
$$F_2\big(w(\mathrm{t}), u(\mathrm{t})\big) = \alpha_2 u(t) - \gamma_2 u^2(t) - \beta_2 w(\mathrm{t})u(t).$$

We have two different cases:

1.
$$\begin{cases} \alpha_1 + \alpha_2 - q > 0 \\ \beta_1 - 2\gamma_2 < 0 \end{cases}. \tag{7}$$

2.
$$\begin{cases} \alpha_1 + \alpha_2 - q < 0 \\ \beta_1 - 2\gamma_2 > 0 \end{cases}. \tag{8}$$

Consider the **first case** (7).

Taking into account (6), the divergence $E\big(u(\mathrm{t}), w(\mathrm{t})\big)$ of the vector field $\vec{F}$ will take the form

$$div\vec{F} \equiv E\left(u(\mathrm{t}), w(\mathrm{t})\right) \equiv\ \alpha_1 + \alpha_2 - q - (2\gamma_1 + \beta_2)w(\mathrm{t}) + (\beta_1 - 2\gamma_2)u(t) \tag{9}$$

and vanishes on the straight line of the phase plane of the solutions

$$w(\mathrm{t}) = \frac{\beta_1 - 2\gamma_2}{2\gamma_1 + \beta_2}u(t) + \frac{\alpha_1 + \alpha_2 - q}{2\gamma_1 + \beta_2}. \tag{10}$$

A line (10) in the first quarter of the phase plane is a segment connecting the points $M_1\left(0, \frac{\alpha_1+\alpha_2-q}{2\gamma_1+\beta_2}\right)$ and $N\left(\frac{\alpha_1+\alpha_2-q}{-\beta_1+2\gamma_2}, 0\right)$ (Fig. 1).

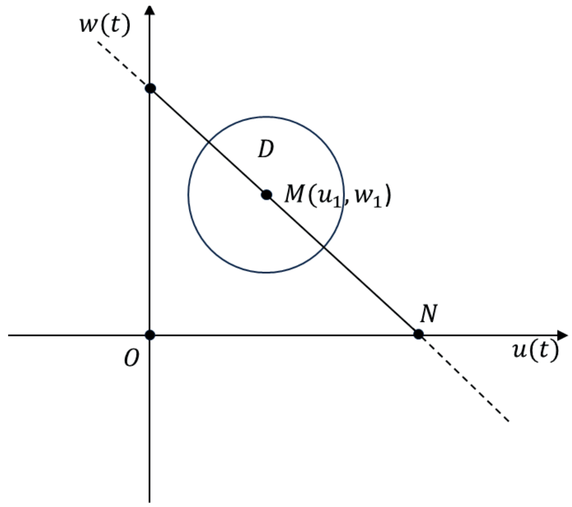

Fig. 1

Now consider the case when the initial conditions (2) satisfy the conditions

$$0 < u_1 < \frac{\alpha_1+\alpha_2-q}{-\beta_1+2\gamma_2}, w_1 = \frac{\beta_1-2\gamma_2}{2\gamma_1+\beta_2}u_1 + \frac{\alpha_1+\alpha_2-q}{2\gamma_1+\beta_2}. \quad (11)$$

The following theorem can now be stated.

Theorem 1. The non-linear system of ordinary differential equations (5), (6) when the (7) is fair and executed (11), in some simply connected domain $D \subset (O, u(\mathrm{t}), w(\mathrm{t}))$ the first quarter of the phase plane $(O, u(\mathrm{t}), w(\mathrm{t}))$ has the solution in the form of the closed trajectory which completely lies in this domain.

Proof. Let's show that the divergence of the vector field $\vec{F}$ according to (9) vanishes on the line

$$\alpha_1 + \alpha_2 - q - (2\gamma_1 + \beta_2)w(\mathrm{t}) + (\beta_1 - 2\gamma_2)u(t) = 0 \quad (12)$$

phase plane of the solutions $(O, u(\mathrm{t}), w(\mathrm{t}))$.

Thus, the vector field divergence in the first quarter with physical content is equal to zero on the a segment connecting the points $M_1\left(0, \frac{\alpha_1+\alpha_2-q}{2\gamma_1+\beta_2}\right)$ and $N\left(\frac{\alpha_1+\alpha_2-q}{-\beta_1+2\gamma_2}, 0\right)$.

Suppose the initial conditions (2) satisfy (11).

It is clear, that $E\big(u(\mathrm{t}), w(\mathrm{t})\big)$ divergence (9) of the vector field $\vec{F}\begin{pmatrix}F_1\\F_2\end{pmatrix}$, in some simply connected domain $D \subset (O, u(\mathrm{t}), w(\mathrm{t}))$, containing the $M(u_1, w_1)$ point lying on the line (10), changes its sign.

Then by Bendixson's theorem, there is a closed integral trajectory of the non-linear system of ordinary differential equations (5)–(7), (11), which lies entirely in this domain $D \subset (O, u(\mathrm{t}), w(\mathrm{t}))$ [19].

Theorem 1 is proved.

Consider the **second case** (8).

In this case, we assume that the constant coefficients (5), (6) satisfy (8).

The divergence of the vector field $E\big(u(\mathrm{t}), w(\mathrm{t})\big)$ is zero on the line (10) of the phase plane $(O, u(\mathrm{t}), w(\mathrm{t}))$ of solutions.

In the case (8) a line (10) passes through the points $M_2\left(0, \frac{\alpha_1+\alpha_2-q}{2\gamma_1+\beta_2}\right)$ and $N_1\left(\frac{\alpha_1+\alpha_2-q}{-\beta_1+2\gamma_2}, 0\right)$. In this case, the first quarter of the phase plane of solutions belongs only to the part of the line (10) with the left end at the point N_1 (Fig. 2).

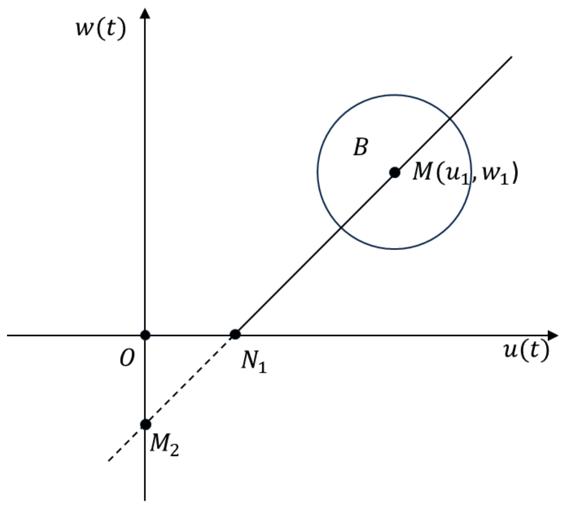

Fig. 2

In the case (8), we will select the initial conditions (2) in such a way that they satisfy the condition

$$u_1 > \frac{\alpha_1+\alpha_2-q}{-\beta_1+2\gamma_2} > 0, w_1 = \frac{\beta_1-2\gamma_2}{2\gamma_1+\beta_2} u_1 + \frac{\alpha_1+\alpha_2-q}{2\gamma_1+\beta_2}. \quad (13)$$

Similarly, the following **theorem 2 is proved.**

Theorem 2. The non-linear system of ordinary differential equations (5), (6) when the (8) is fair and executed (13), in some simply connected domain $B \subset (O, u(\mathrm{t}), w(\mathrm{t}))$ the first quarter of the phase plane $(O, u(\mathrm{t}), w(\mathrm{t}))$ has the solution in the form of the closed trajectory which completely lies in this domain.

Proof. Let's show that the divergence of the vector field $\vec{F}$ according to (9) vanishes on the line (10) phase plane of the solutions $(O, u(\mathrm{t}), w(\mathrm{t}))$.

In this case, the first quarter of the phase plane of solutions belongs only to the part of the line (10) with the left end at the point N_1.

Suppose the initial conditions (2) satisfy (13).

It is clear, that $E\big(u(\mathrm{t}), w(\mathrm{t})\big)$ divergence (9) of the vector field $\vec{F}\begin{pmatrix}F_1\\F_2\end{pmatrix}$, in some simply connected domain $B \subset (O, u(\mathrm{t}), w(\mathrm{t}))$, containing the $M(u_1, w_1)$ point lying on the line (10), changes its sign.

Then by Bendixson's theorem, there is a closed integral trajectory of the non-linear system of ordinary differential equations (5), (6), (8), (13) which lies entirely in this domain $B \subset (O, u(\mathrm{t}), w(\mathrm{t}))$ [19].

Theorem 2 is proved.

Conclusion. Computer modeling of the first stage for various functions of coefficients gives two qualitatively various pictures, but both at the end of the first stage (2500 BC) yield approximately identical result, the Common-Kartvelian-speaking populace in this century became the 3–3.5 million. The qualitative difference is that: in the first case, the Common-Kartvelian-speaking populace always grows from 1 million to 3–3.5 million; in the second case: the Common-Kartvelian-speaking populace at first it grows to 4–4.5 million (maximum development and economic situation), then, due to certain reasons (perhaps there was not enough land, food, etc.), it began to decrease to 3–3.5 million.

The second stage of transformation of the Common-Kartvelian-speaking populace is described by two different mathematical models. The first part of the Common-Kartvelian-speaking populace emigrated to Europe and gradually completely or partially assimilated on the European continent. An exact analytical solution was found. Cases of complete or partial Common-Kartvelian-speaking populace assimilation researched.

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აბსტრაქტი. ნაშრომში განხილულია საერთოქართველურ ენაზე მოლაპარაკე ხალხის ტრანსფორმაციის ორი ეტაპი: პირველი ეტაპი იყო ძვ. 5000-2500 წწ., მეორე

ეტაპი – ძვ.წ. 2500–1000 წწ. მეორე ეტაპის დროს პროტოქართველური მოსახლეობიდან ჩამოყალიბდა სამ შტოდ. ესენია: სვანური, კოლხურ-ქართული და პელასგური. ეს უკანსაკნელი გადასახლდა ევროპის კონტინენტის სხვადასხვა რეგიონში. პირველი ეტაპისთვის კომპიუტერული მოდელირება გამოიყენება

რიკატის დიფერენციალური განტოლების ცვლადი კოეფიციენტების შემთხვევაში და მიღებულია შესაბამისი რიცხვითი ამოხსნები. მეორე ეტაპი აღწერილია ორი განსხვავებული მათემატიკური მოდელით. საერთოქართველურ ენაზე მოსაუბრე ხალხის ერთი ნაწილი, პელასგები წავიდნენ ევროპაში და დაიწყო მათი ნაწილობრივი ან სრული ასიმილაციის პროცესი ევროპის კონტინენტზე. კოლხურ-ქართულ და სვანურ ენებზე მოლაპარაკე მოსახლეობა დამკვიდრდა სამხრეთ კავკასიასა და ნაწილობრივ ანატოლიაში. ამ ორ ენაზე მოლაპარაკე ხალხებს შორის ურთიერთქმედების პროცესის აღსაწერად წარმოდგენილია ჩვეულებრივი დიფერენციალური განტოლებების ორგანზომილებიანი არაწრფივი სისტემა ცვლადი კოეფიციენტებით. კერძო შემთხვევაში განხილულია ჩვეულებრივი დიფერენციალური განტოლებების ორგანზომილებიანი სისტემა მუდმივი კოეფიციენტებით. სისტემის მუდმივ კოეფიციენტებს შორის გარკვეული ურთიერთდამოკიდებულების ორ შემთხვევაში ნაჩვენებია უცნობი ორკომპონენ-ტიანი ვექტორ-ფუნქციის დივერგენციის ფაზური სიბრტყის პირველ მეოთხედში ნიშანცვლადობის შესახებ ნახევარწრფეზე გადასვლისას. ბენდიქსონის პრინციპის გათვალისწინებით, დამტკიცებულია თეორემები ვექტორული ველის დივერ-გენციის ნიშანცვლადობისა და შეკრული ინტეგრალური წირების არსებობის შესახებ ცალბმულ არეში, რომელიც შეიცავს ნახევარწრფეზე მდებარე წერტილს (ტრაექტორიის საწყისი წერტილი).

ამრიგად, მათემატიკურმა ანალიზმა მუდმივი კოეფიციენტების შემთხვევაში აჩვენა, რომ კოლხურ-ქართული მოსახლეობის მიერ სვანური მოსახლეობის სრული ასიმილაცია არ ხდება და ეს ორი მოსახლეობა ერთ ტერიტორიაზე მშვიდობიანად თანაარსებობს.

საკვანძო სიტყვები: *მათემატიკური მოდელირება, დინამიური სისტემა, კოლხურ-ქართული და სვანური მოსახლეობები, ბენდიქსონის პრინციპი.*